Factors and Multiples

FACTOR: A **Factor** of a whole number is a whole number that can be multiplied by another whole number, to give a product of the original number.

A factor is also called a **divisor** because it can be divided evenly into the original number.

Example: 25 is a factor of 100 because : 25 x 4 = 100.

4 is also a factor of 100. (4 x 25 = 100)

25 and 4 are **factor pairs.**

Product: The result of multiplication. The PRODUCT of 5 and 10 is 50

A **Proper Factor** is any factor of a number besides the number itself. (1 is sometimes excluded also). The proper factors of 12 are: 2,3,4, and 6.

*Factors "GO INTO" the number, and are **always** smaller or equal to the number you are working with.*

MULTIPLE: A Multiple of a number is the product of that number and another whole number.

Example: 24 is a multiple of 12 since 12 x 2 = 24.

24 is also a multiple of 2 since 2 x 12 = 24.

You get a MULTIPLE by MULTIPLYING. Multiples are always Greater to or Equal to the original number.

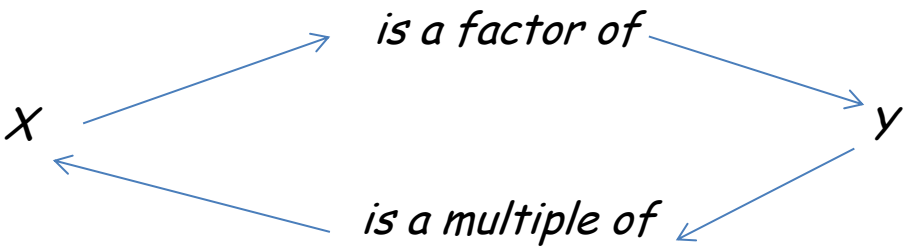
Example:

Factors of 100

1,2,4,5,10,20,25,50,100

Multiples of 100

100,200,300,400,500…….



Prime Numbers have no factors other than 1 and the number itself. Prime numbers have NO PROPER FACTORS (except 1, which is sometimes excluded)

Prime numbers start with **2**. Other Prime numbers are:

3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59…

Prime numbers are not fractions, decimals, or negatives. 0 and 1 are not Prime either.

Composite Numbers DO have other (proper) factors.

12 is Composite since it has proper factors (like 6).

- Negative numbers, fractions, and decimals are not considered prime or composite

Square Numbers are the result of multiplying the same whole number by itself. (This is called squaring the number)

For example: 36 is a square number since 6 x 6 = 36

Also, we would say $6^2 = 36$ (6 squared = 36)

First 15 Square Numbers:

1,4,9,16,25,36,49,64,81,100,121,144,169,196,225

Divisibility Rules:

A Number is divisible by:

2: if the last digit is even

3:if the sum of the digits is divisible by 3

4: if the last two digits are divisible by 4

5: if the last digit is 0 or 5

6: if the number is divisible by 2 AND 3

9:if the sum of the digits is divisible by 9

10: if the last digit is zero

GCF and LCM

Common Factors: Are factors that are shared by two or more whole numbers.

1,2,4,and 8 are common factors of 24 and 32

The **GREATEST COMMON FACTOR (GCF)** is the greatest factor shared by two or more whole numbers

8 is the GCF of 24 and 32

Common Multiples: Are multiples that are shared by two or more whole numbers.

96 and 192 are common multiples of 24 and 32.

The **LEAST COMMON MULTIPLE (LCM)** is the least multiple that is shared by two or more whole numbers.

96 is the LCM of 24 and 32.

*There is never a LEAST common factor, because that is always 1!!!

*There is never a GREATEST common multiple, because multiples go on forever! (the list is infinite)

Number Rules:

Exponents:

4^3 4 is the BASE, 3 is the EXPONENT

You multiply the base by itself the number of times the exponent tells you

Ex: $4^3 = 4 \times 4 \times 4 = 64$

Distributive Property:

$a (b + c) = ab + ac$

ex: $4 (3+ 7) = 4(3) + 4(7)$

$4(10) = 12 + 28$

$40 = 40$

Order of Operations:

Remember PEMDAS

P. Parenthesis. Do whatever is in parenthesis first.

Parenthesis could be (),{},[],||

E. Exponents. Evaluate any exponents second.

Exponents could be like 4^3 or like $\sqrt{7}$

M.D. Multiplication and Division: Simplify any multiplication or division in the order they appear (left to right)

A.S. Addition and Subtraction: Simplify any addition or subtraction last, in the order they appear (left to right)

Ratios: A ratio is a comparison of two quantities.

Can be expressed 3 ways:

a) In WORDS: " *to", "for every", "for each" , "per"*

b) COLON :

c) As a FRACTION

Example: There are 14 boys and 16 girls in class 213. The ratio of boys to girls in 213 is 14 to 16. This can simplify to 7 to 8.

The ratio of boys to girls is 7:8 or 7/8.

- Order matters!
- Don't put units with ratios
- Can compare:

<u>Part to Part or Part to Whole</u>

Example: The ratio of boys to girls is 7:8. This is PART to PART. The ratio of boys to the whole class is 7:15. This is PART to WHOLE

Ways to work with ratios:

A) Equivalent Fractions (equivalent ratios) or **PROPORTIONS**

- can simplify the fraction
- can find L.C.D.
- can cross multiply

  ex:

B) Table

- Can count "up" with addition
- Can "skip" by multiplying.

  Ex.

Blue Paint	2	4	6	8
Yellow Paint	3	6	9	12

C) Find the UNIT RATE:

Unit rate is a rate out of ONE.

Unit rate words:

"per" "for every one" "for each one"

Examples:

30 miles PER hour (30 miles per 1 hour),

1.5 gallons of yellow paint for EACH gallon of blue

D) Tape Diagram

E) Draw a picture

Rates:

A RATE is a ratio with different units

Example:

4 shells for $1

8 shells for $2

4 cookies for every 5 students

5 miles per hour

10 miles for every 2 hours

Key words for rates:

"for" " for each" "for every" "PER"

A UNIT RATE is a useful and helpful rate because it is a ratio out of 1!!!

Examples of unit rates:

5 MILES PER HOUR (5 MILES FOR EVERY ONE HOUR)

4 shells for 1 dollar

- Can find the unit rate by
  - Dividing
  - Making a table
  - Proportions (equivalent ratios)
- To figure out, make the unit quantity ONE!
- Unit rates help us find out ALL OTHER EQUIVALENT RATIOS!!

UNIT RATE:

The unit rate is a rate OUT of ONE.

You can find the unit rate by:

A) Dividing :

Example: A car drives 90 miles in 3 hours.

90/3 means 90 ÷ 3, which is 30, so the unit rate is 30 miles per ONE hour

B) Going "backwards" in a table (using division or subtraction) until you reach 1, then scaling forward (using multiplication or addition).

Example:

Cups of sugar	2.5	5	10	15	20	25	30
Cups of flour	1	2	4	6	8	10	12

The unit rate is 2.5 cups of sugar for each cup of flour

C) Setting up a proportion (equivalent ratio) where one fraction has a 1 in the correct spot.

Example:

Jacob reads 21 pages in his book in 3 days. His unit rate is:

$\frac{21 \text{ (pages)}}{3 \text{ (days)}} = \frac{? \text{ (pages)}}{1 \text{ (day)}}$

Cross multiply to see 21 x 1 = 21, and 3 x 7 = 21, so the unit rate is 7 pages PER DAY.

- <u>There can be 2 Unit rates for a situation.</u> **(a to b and b to a)**
  example: Jacob reads 7 pages in 1 day.(7 pages per day.)
  He also spends 1/7 of a day per page

  <u>Rate Formats:</u>

  A Rate → Quantity A out of Quantity B
  Or A:B

  For unit rate
  The B spot becomes 1 by making it
  $\frac{A}{B} : 1$

  The "OTHER" unit rate is B: A or $\frac{B}{A} :1$

  Example: 3 blocks in 4 minutes becomes $\frac{3}{4}$ *blocks per minute.*
  The "other " unit rate is 4 minutes for 3 blocks, or *4/3 minute per block*

  Rates:
  A:B means A ÷ B or A/B

  UNIT PRICE: is always Price per 1 unit ($4 per ounce)

  UNIT SPEED: is always distance per 1 unit of time (30 Miles per hour)

Operations with Fractions:

For all operations, convert mixed numbers into improper fractions

To **ADD or SUBTRACT:**

- ➔ Find Least common Denominator
- ➔ Add/Subtract numerator
- ➔ Keep denominator
- ➔ Simplify if possible

To **Multiply:**

- ➔ Multiply straight Across
- ➔ Simplify if possible

To **Divide:**

- ➔ "flip" the 2nd fraction (reciprocal)
- ➔ Then multiply like normal
- ➔ Simplify if possible

Operations with Decimals:

To **ADD or SUBTRACT:**

➔ Line up decimal point, then add or subtract like normal.

➔ Remember to bring decimal point down

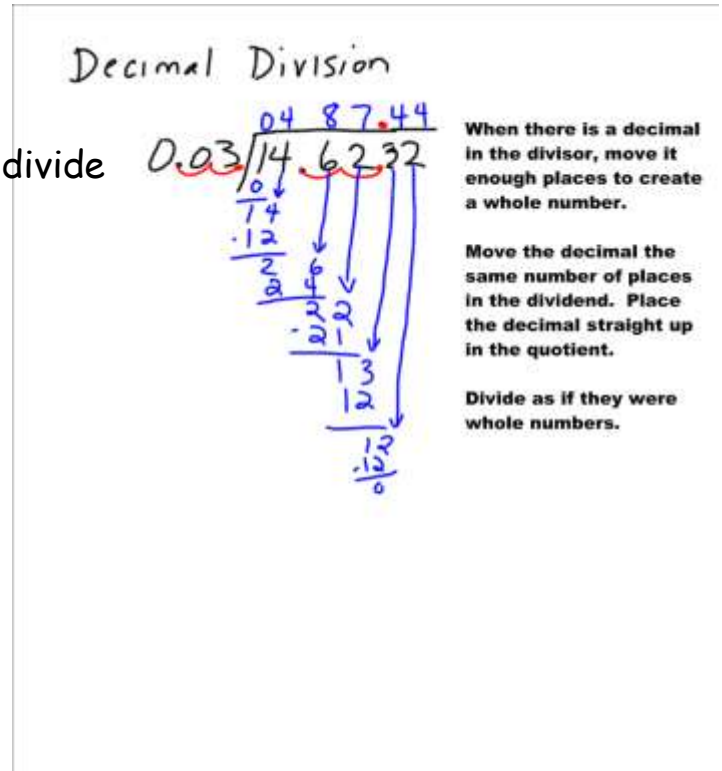
To **Multiply**

➔ Multiply like normal first

➔ Put back decimal by counting the number of decimal spaces in the 2 numbers you multiplied

To **Divide:**

➔ Move Decimal like shown, then divide



Rational Numbers:

Rational Numbers: are numbers that can be turned into fractions. (Whole numbers, integers, fractions, and some decimals are all rational numbers)

Ex: 0, 2 , -4, 5.66 , $\frac{1}{2}$, $\frac{3}{4}$ 14.94, -9.1 are Rational numbers

Integers: are the whole numbers and their opposites (positive and negative whole numbers)

Ex: … -3, -2, -1, 0 , 1, 2, 3,…. are integers

Positive Numbers: are to the right of 0 on a number line

Negative Numbers: are to the left of 0 on a number line

Opposites: 2 numbers are opposites if they are the same distance from ZERO.

-3 and 3 are opposites. They are both 3 spaces from 0.

Symbol : is – The opposite of (4) can be written as – (4)

The opposite of (-5) can be written as – (-5) which is the same as 5

Absolute Value: is a number's distance from 0.The symbol is | | . Example, | -3 | means the absolute value of -3, which is 3.

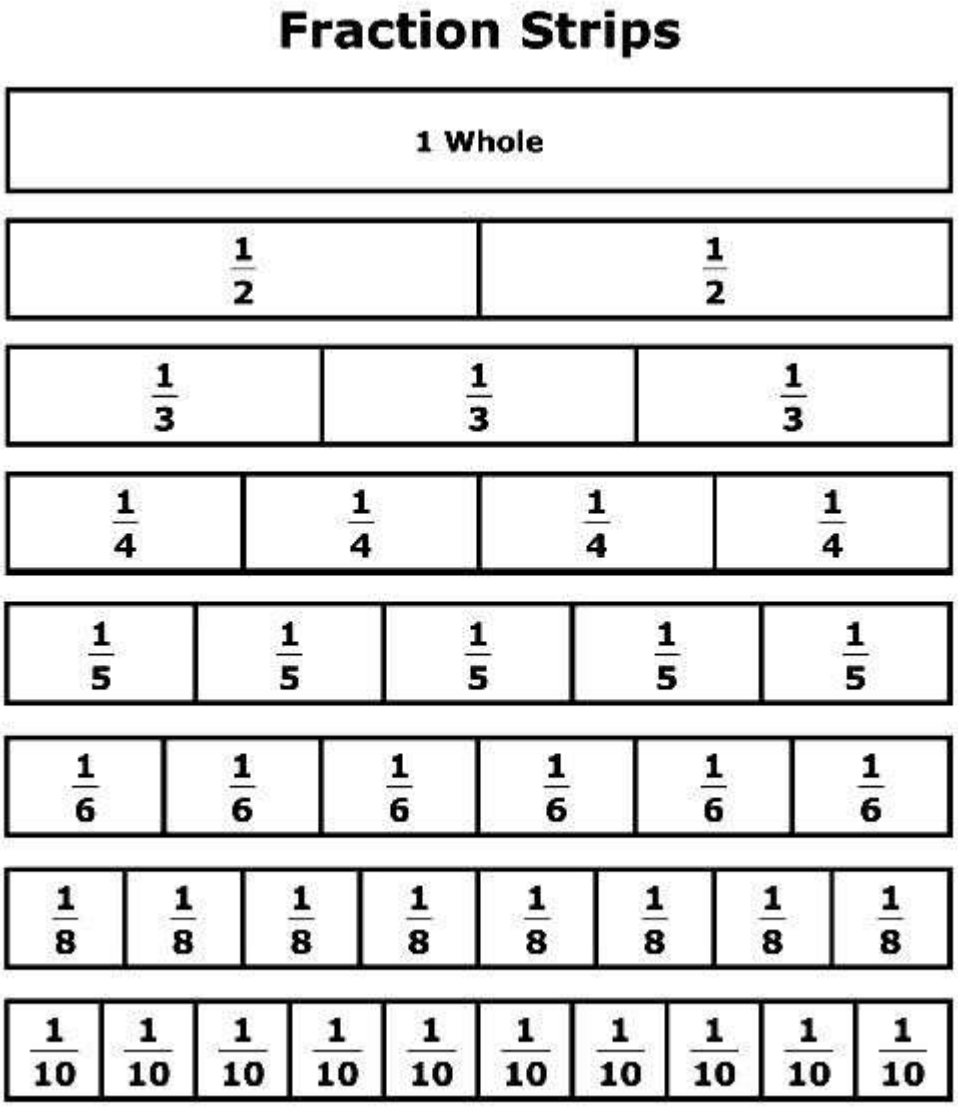
| -4 | is 4. | 5 | is 5

Benchmark Fractions and Decimals and Percents

Common Fraction and Decimal Equivalents

Fraction	Decimal	Percent
1	1.0	100%
1/3	.333	33.3%
1/4	.25	25%
1/5	.2	20%
1/8	.125	12.5%
1/10	.1	10%
1/2	.5	50%
2/3	.666	66.6%
3/4	.75	75%
2/5	.4	40%
5/8	.624	62.5%
9/10	.9	90%

Fraction Strips



Strategies for finding and placing Rational Numbers on a Number Line

- Place positive number on number line, then do the opposite for the negative.

- Use Benchmarks as reference points, to place the number in between.
  - Example: what 2 integers is the number between?
  - Is greater or less than $\frac{1}{2}$ way between?
  - Is it greater or less than $\frac{1}{4}$?

- Convert all fractions to decimals or all decimals to fractions

- Find a common denominator (or numerator) for all fractions

- Cross multiply fractions to see which is greater

- Break up the number line using the LCD of the fractions

Comparing Numbers:

Inequalities: show 2 statements that are NOT EQUAL. Use symbols:

< "is less than" *5 < 9 (5 is less than 9)*

> " is greater than" 6 > 4 (6 is greater than 4)

≤ " is less than or equal to" *x ≤ 4 means everything less than 4, including 4*

≥ " is greater than or equal to" *x ≥ 5 means everything greater than 5 including 5*

Real life Number Lines:

0 on a number line can represent:

"Sea Level", a $0 bank balance (you have no money and owe no money)

No distance travelled, a starting point,

Positive Numbers represent: Gains, moving forward, above sea level, a positive bank balance (you have money saved)

Negative Numbers represent: Losses, moving backwards, below sea level, a negative bank balance (you owe money, or are in debt)

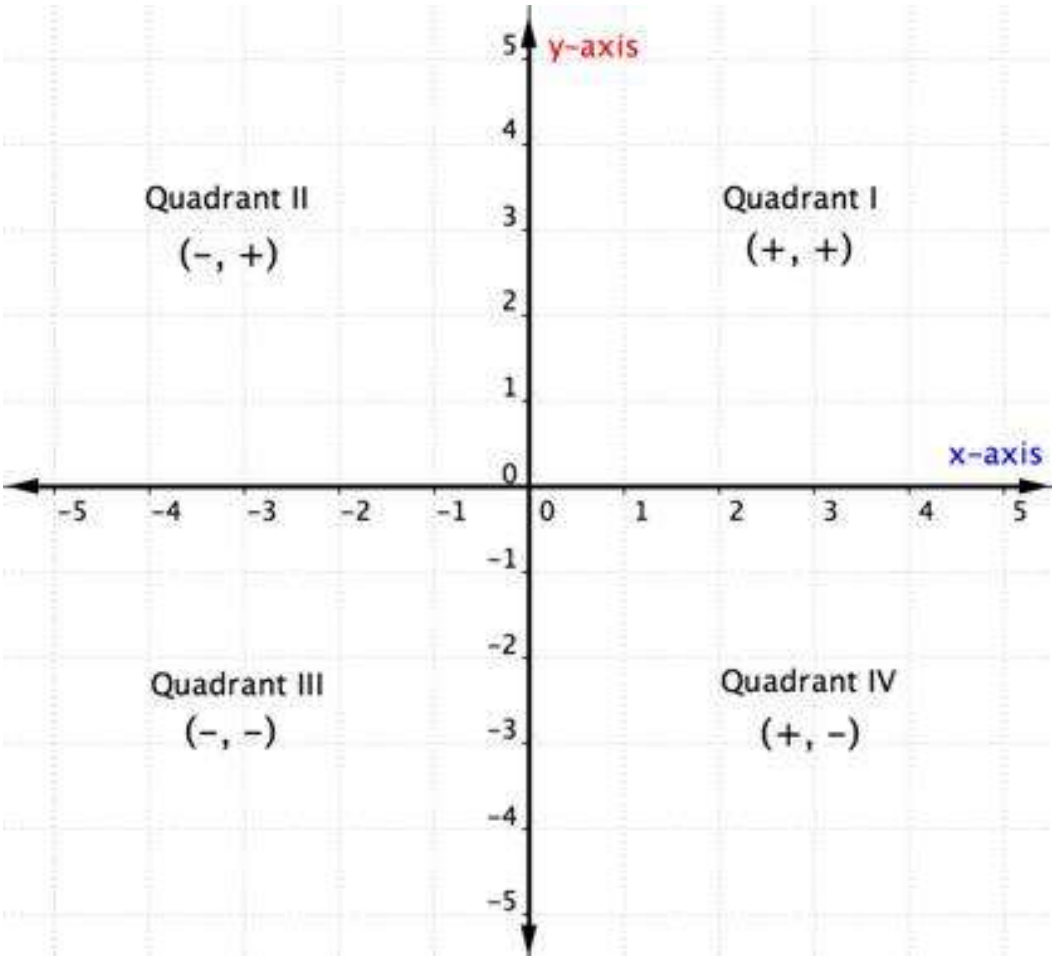
You are moving **left** (or down) a number line when the value is **DECREASING**

- ➔ Drops, decreases, goes down, gets colder, goes below, goes backwards, losing or spending money, traveling down

You are moving **right** (or up) on a number line when the value is **INCREASING**

- ➔ Raises, Rises increases, goes up, gets warmer, goes above , goes forward, earning money, traveling up

Coordinate Graphing



To Plot a Point:

(x, y) means you move x spaces right or left and y spaces up or down

If x is + you move right, if x is – you move left

If y is + you move up, if y is – you move down

(Walk Right or Left, Climb Up or Down)…. You walk before you climb

Always start at the origin (0,0)

Reflections on the Coordinate Plane :

To Reflect over the *y axis*, the y coordinate stays the same and the x coordinate changes to it's opposite.

To Reflect over the *x axis*, the x coordinate stays the same and the y coordinate changes to it's opposite.

Working with Polygons

Perimeter: is the distance around the outside of a shape.

Area: is the space inside

Basic Polygons: *(straight edged shapes with 3 or more closed sides)*

Triangle : 3 sides

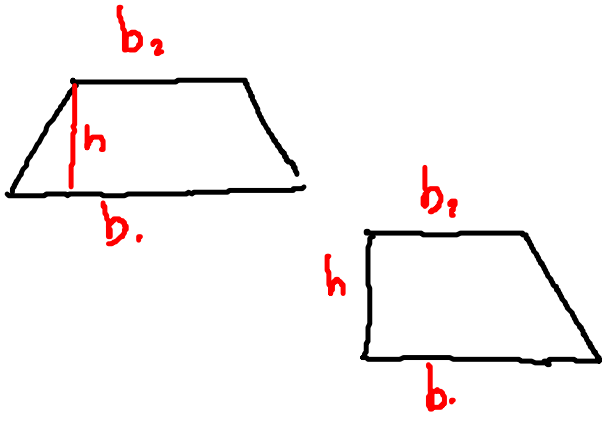
Area = $\frac{1}{2}$ b*h or $\frac{b \cdot h}{2}$

Quadrilaterals : 4 sides

(some quadrilaterals are squares,rectangles,trapezoids,parallelograms, rhombi)

Area of a square , rectangle, parallelogram = b*h

Area of a trapezoid = $\frac{1}{2}$ h (b1 + b2)



DIVISION: Divides a "Starting" number into pieces or groups.

Dividend ÷ Divisor = Quotient

(What you start with) ÷ (size of each Group) = (Number of groups)

(What you start with) ÷ (Number of groups) = (size of each group)

When you divide in the opposite order you get the Reciprocal (flipped fraction)

Remember

Division ←→ Fraction

4 ÷ 7 = 4/7

7÷4= 7/4

Dividing by a fraction is the same as Multiplying by its reciprocal.

(Remember **Keep Change Flip**)

Keep the first number

Change Division to Multiplication

Flip the 2nd fraction to its reciprocal

MATH ← → English Translation Guide

+	-	x
Plus *Sum* *Add* *Added with* *Added to* *Increased* *Greater* ***than*** *More* ***than*** *Combined* *total*	*Minus* *Difference* *Subtract* *Subtracted* ***from*** *Less* ***than*** *Take away* *Decreased by* "than" and "from" the order flips	*(4 x 3)* *4*3* *(4)(3)* *4(3)* *Times* *Multiply* *Product* *Of* *Doubled (x2)* *Tripled(x3)* *Halved (x $\frac{1}{2}$)*
÷	=	x, y , n ….
Divided *Quotient* *Per* *For every* *Split into* *For each* *Into* *Shared* *Equal groups* *Out of (fraction)*	*Equals* *All together* *Totals* *Is* *Will be* *Will become* *result*	Are variables (letters that replace numbers) *A number* *An unknown number* *An unknown amount*

Expressions and Equations:

An **EXPRESSION** is a number sentence.

Can Contain Numbers, Variables, and Operation (symbols)

Ex. 2, 5x, 7x + 5, $\frac{1}{2}$ n -4

An **EQUATION** is 2 expressions that are **Equal!**

Ex: 3x = 15, 5y = 90, 3+ 3 = 6

A **Variable** represents an unknown number. It can vary

A **Constant** does not vary. It is a number

A constant next to a variable (being multiplied) is called a **coefficient.**

Ex: 4x + 7

A **TERM** is a number, variable, or combination of numbers and variables connected with multiplication or division.

ADDITION or SUBTRACTION separates terms

Ex: 4, x, 5x, 12y 9 (all single terms: MONOMIALS)

12a + 3b (has 2 terms: BINOMIAL)

3a + 2a (can COMBINE to make 5a)

To **Evaluate** an expression, you SUBSTITUTE a number in for the variable . (follow PEMDAS)

To check if an equation is true, SUBSTITUTE for the variable and check.

Equation Format:

A LINEAR(makes a straight line) equation will look like:

$$Y = mx + b$$

y is the : Dependent variable

output

"answer"

x is the : independent variable

input

control

m is the: coefficient

constant rate (sometimes also the unit rate)

constant rate of change

slope (8th grade)

b is the : constant

initial value

head start

y intercept (8th grade)

Making a graph:

- Choose appropriate scale (what number are you "going by"?)
    - The smaller the scale ,the better. Make sure it fits on paper first

- Scale must be consistent (the same!)
- Label the x and y axis, and put arrows at end
- Give the graph a title
- Label the units on the x and y axis
    - Remember, x is the independent variable, and y is the dependent variable

- Plot the points, and connect if necessary

Independent variable: is the one you control, the one you can choose the value or points you want to measure. The one not affected by the other. (time is always independent)

Dependent variable: is the one that depends on the independent variable. The one that is the result of the experiment or what you are trying to measure/observe.

Solving Equations:

You need to work *backwards* to "undo" what ever was "done". To work backwards you perform the **inverse operations** to go "back" to the original number.

Inverse operations:

Addition←→ Subtraction

Multiplication ←→ Division

You need to undo every operation that was done by performing the inverse operation in the opposite order that they were originally performed. (reverse PEMDAS).

$$\begin{aligned} 9x + 3 &= 21 \\ -3 & \quad -3 \\ \frac{9x}{9} &= \frac{18}{9} \\ x &= 2 \end{aligned}$$

Inequalities: compare 2 expressions that are not equal. Use the following symbols:

> (is greater than)

< (is less than)

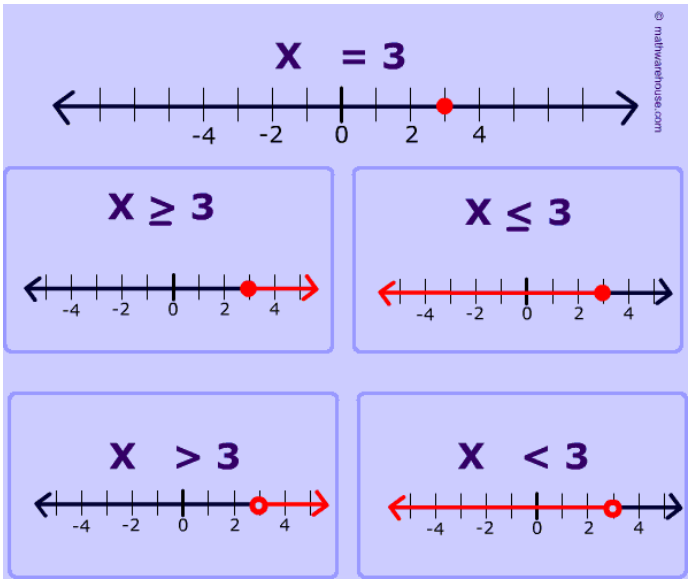
≥ (is greater than or equal to)

≤ (is less than or equal to)

You "solve" like an equation, but you keep the symbol instead of the = sign. There are infinite solutions!

To **Graph an inequality:**

1) "solve"
2) Circle solution
   a. Open circle o for < and >
   b. Closed circle● for ≤ and ≥
3) Shade number line left or right
   a. Left for less than <,≤
   b. Right for greater than >, ≥

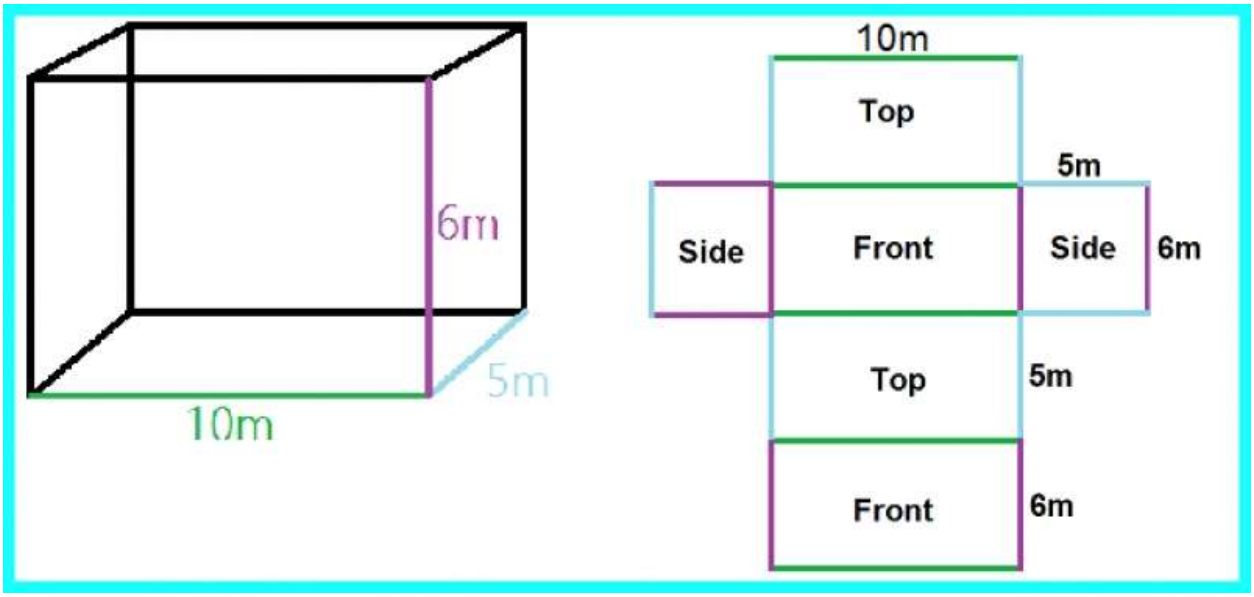


Volume: is the space inside a 3 dimensional figure.

Volume of ANY PRISM: is the (area of the base) x height

Ex: a box (rectangular prism) Volume is: length x width x height

Surface Area: is the area of ALL surfaces of any 3d object. Just find the area of each surface (face) separately and add up!



PERCENTS %

A Percent is a ratio out of 100

PER/CENT

20 Percent (20 %) means 20 out of 100 (20/100)

Ways to work with PERCENTS:

1) Proportion (2 equivalent ratios or fractions).

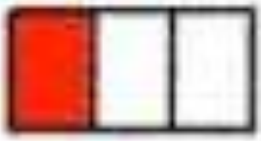
$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{percent}}{100}$$

2) Convert the percent to a decimal.

(move the decimal point 2 places to the left to convert a percent to a decimal. Ex. 52% = 0.52)

- Remember: "OF" means to multiply
  - 30% of 70 means 0.30 x 70 , which is 21

Fractions, Decimals, & Percents

Fraction	Decimal	Percent	Picture
$\frac{1}{10}$	0.1	10%	
$\frac{1}{5}$	0.2	20%	
$\frac{1}{4}$	0.25	25%	
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}\%$	
$\frac{1}{2}$	0.5	50%	
$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}\%$	
$\frac{3}{4}$	0.75	75%	
1	1.00	100%	

STATISTICS:

Measures of Central Tendency:

These are statistical terms that try to figure out the "center" of the numbers that are in a group of data.

Three **M**easures of central tendency:

Mean: (the average. Add up all #s and divide by how many there are)

Median (the middle. Place the data in order from least to greatest and find the data point in the middle)

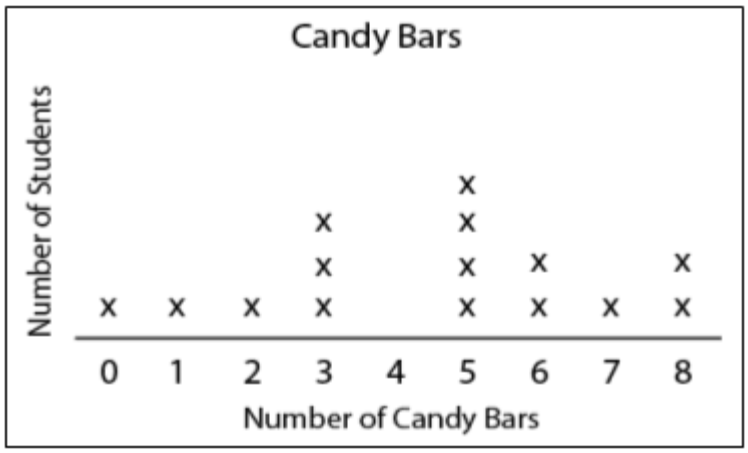
Mode (the most common or most popular. The data point with the highest frequency. There could be no mode, or there could be more than one mode)

An OUTLIER- is a number in a set of data that is much bigger or smaller than the rest of the numbers

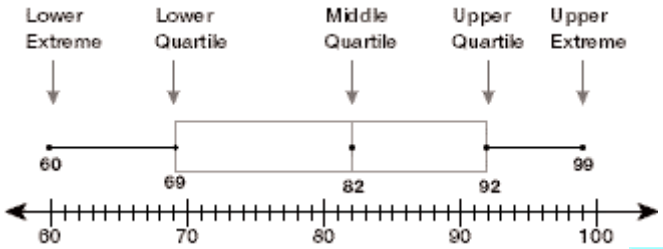
The **range** is the distance from the highest to the lowest data point (subtract the highest – lowest)

Different ways to illustrate data:

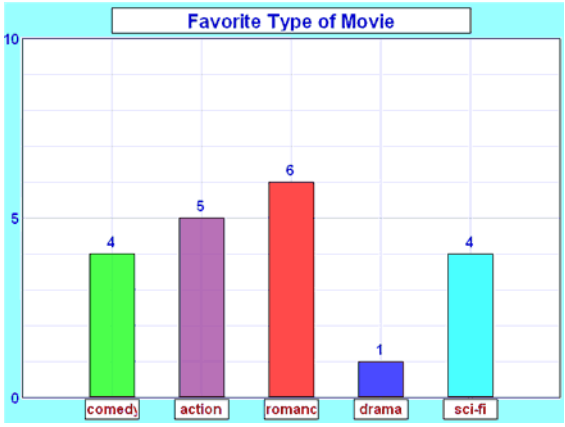
Line Plot or Dot Plot



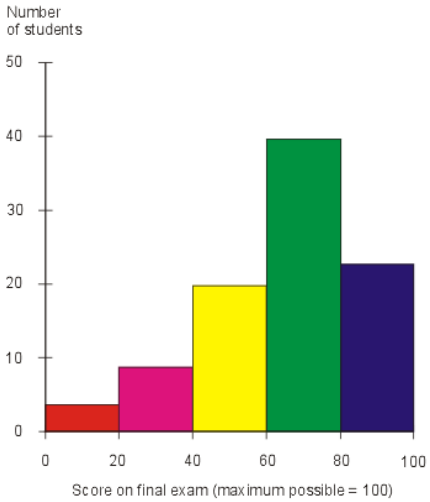
Box and whisker Plot



Bar Graph

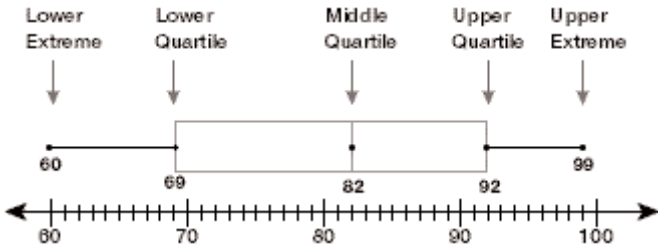


Histogram



Creating a box and whisker plot and Calculating the Inter Quartile Range (IQR):

1) Write the data in order (smallest to largest)
2) Find the median
3) Divide the data into quartiles by finding the median of the data to the left of the median, and the median of the data to the right of the median
    a. These are the Lower Quartiles (quartile 1) and Upper Quartile (quartile 3)
4) Identify the lower extreme and the upper extreme



The IQR is the distance from Q1 to Q3 . Subtract the Lower Quartile from the Upper quartile. 50 percent of the data is in this middle piece

Mean Absolute Deviation: (M.A.D.)

The MAD is the average distance each point is from the average.

1) Find the mean
2) Subtract each data point from the mean
3) Find the mean using the numbers in step 2.

The MAD and IQR tells you how much variability the data has. The higher the MAD and/or IQR, the more spread out and variable the data is.

Bar Graphs and Histograms

Bar Graphs: show (usually) discreet data. (data that is not connected). The bars do not touch, and there is usually not a specific order the bars must be placed.

Histograms: Show continuous data (think history). The bars touch because one bar leads into the other. The bars are usually in intervals.(like 0-4 years old, then 5-9 years old…). The intervals must all be equal in length .
From a histogram , you can also see the distribution and any variability.